On the Irreducible BRST Quantization of Spin-5/2 Gauge Fields

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Abstract
Spin-5/2 gauge fields are quantized in an irreducible way within both the BRST and BRST-anti-BRST manners. To this end, we transform the reducible generating set into an irreducible one, such that the physical observables corresponding to these two formulations coincide. The gauge-fixing procedure emphasizes on the one hand the differences among our procedure and the results obtained in the literature, and on the other hand the equivalence between our BRST and BRST-anti-BRST approaches.
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1 Introduction
The power of the antifield BRST formalism [1]-[4] has been fully proved lately. This approach can be applied to both irreducible and reducible theories. A more symmetrical treatment is given by the antifield BRST-anti-BRST method [5]-[8]. Although less important than the BRST symmetry, the BRST-anti-BRST procedure helps at a correct understanding of the non-minimal sector. The non-minimal variables are particularly important when

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dealing with redundant systems, being required during the gauge-fixing pro-
cess. A typical class of reducible models are free massless higher spin gauge
fields [17]-[28]. Such theories are important due to their connection with
string theory, and, because of their remarkable gauge symmetries, they are
promising candidates for building a unified physical theory. In the mean-
time, the existence of a large class of nontrivial interacting higher spin gauge
theories [29], at least in four dimensions, reveal the necessity of investigating
this type of models.

In this paper we quantize free massless spin-5/2 gauge fields. Although
first-stage reducible, we show that this model can be consistently approached
in an irreducible manner following both antifield BRST and BRST-anti-
BRST lines. As far as we know, there has not been published such a proce-
dure. Our analysis mainly consists in: (i) replacing the reducible generating
set of the original gauge symmetries with an irreducible one, and (ii) quantiz-
ing the irreducible theory. The irreducible model is obtained by introducing
a spin-1/2 gauge field such that the physical observables arising from the
reducible, respectively, irreducible situation are the same. We mention that
the idea of replacing the reducible symmetry by an irreducible one acting on
new variables is not new. In fact, it originates in the Hamiltonian formal-
ism, where a reducible set of first-class constraints can be replaced with an
irreducible one via introducing some new variables [30]-[31].

The paper is structured in five sections. In section 2, we give a brief
description of the model under study. Sections 3 and 4 are devoted to the
irreducible BRST, respectively, BRST-anti-BRST quantization. Section 5
presents some final conclusions.

2 Spin-5/2 gauge fields

We start with the Lagrangian action [18], [23]

\[ S_0^L [\psi_{\mu\nu}] = \int d^4x \left( -\frac{1}{2} \bar{\psi}_{\mu\nu} \partial \psi_{\mu\nu} - \bar{\psi}_{\mu\nu} \gamma_{\nu} \partial \gamma_{\lambda} \psi_{\lambda\mu} + 2 \bar{\psi}_{\mu\nu} \gamma_{\nu} \partial \lambda \psi_{\lambda\mu} + \frac{1}{4} \bar{\psi}_{\lambda\lambda} \partial \psi_{\mu\mu} - \bar{\psi}_{\lambda\lambda} \partial_{\mu} \gamma_{\nu} \psi_{\mu\nu} \right), \]  

(1)
where $\psi_{\mu\nu}$ is a symmetric Majorana spin tensor. In the sequel, we work with the Pauli metric ($\mu = 1, 2, 3, 4$) and Hermitian $\gamma$-matrices satisfying
\begin{equation}
\gamma_\mu \gamma_\nu + \gamma_\nu \gamma_\mu = 2 \delta_{\mu\nu}. \tag{2}
\end{equation}
Action (1) is invariant under the gauge transformations
\begin{equation}
\delta \psi_{\mu\nu} = (\delta_{\nu\beta} \partial_\mu + \delta_{\mu\beta} \partial_\nu) \left( \delta_{\beta\alpha} - \frac{1}{4} \gamma_{\beta\gamma} \gamma_\alpha \right) \epsilon_\alpha \equiv Z_{\mu\nu\alpha} \epsilon_\alpha, \tag{3}
\end{equation}
with $\epsilon_\alpha$ independent gauge parameters. The transformations (3) are first-stage reducible
\begin{equation}
Z_{\mu\nu\alpha} Z_\alpha = 0, \tag{4}
\end{equation}
with the reducibility functions
\begin{equation}
Z_\alpha = \gamma_\alpha. \tag{5}
\end{equation}
This completes the classical Lagrangian analysis.

3 The irreducible BRST treatment

In this section we develop an irreducible antifield BRST approach for the spin-5/2 gauge fields. Initially, we transform the reducible gauge generators $Z_{\mu\nu\alpha}$ into some irreducible ones. To this end, we associate a spin-1/2 field, $\varphi$, with the reducibility relation (4) and impose its gauge transformation as
\begin{equation}
\delta_\epsilon \varphi = A_\alpha \epsilon_\alpha, \tag{6}
\end{equation}
with $A_\alpha$ some matrices (that may involve the fields) taken to fulfil
\begin{equation}
\det (A_\alpha Z_\alpha) \neq 0. \tag{7}
\end{equation}
From (3) and (4) one can easily see that a possible choice reads as
\begin{equation}
A_\alpha = 1 \partial_\alpha, \tag{8}
\end{equation}
because $A_\alpha Z_\alpha = \partial$ has the inverse $\partial/\Box$. In this way, (3) become
\begin{equation}
\delta_\epsilon \varphi = \partial_\alpha \epsilon_\alpha. \tag{9}
\end{equation}
Next, we investigate the theory described by the action

\[ S^L_0[\psi_{\mu\nu}, \varphi] = S^L_0[\psi_{\mu\nu}], \tag{10} \]

subject to the gauge transformations (3) and (9). A noteworthy feature of this theory is that its gauge transformations are irreducible on account of (7). It is remarkable that the physical observables corresponding to the irreducible, respectively, reducible models coincide. This can be seen as follows. Let \( F(\psi_{\mu\nu}, \varphi) \) be an observable of the irreducible theory. Then, its gauge variation should vanish (at least when the equations of motion hold). This implies

\[ \frac{\delta F}{\delta \psi_{\mu\nu}} Z_{\mu\nu\alpha} + \frac{\delta F}{\delta \varphi} A_\alpha = 0. \tag{11} \]

Multiplying (11) by \( Z_\alpha \), and using (4) and (7) we find

\[ \frac{\delta F}{\delta \varphi} = 0. \tag{12} \]

From (11) and (12) we obtain

\[ \frac{\delta F}{\delta \psi_{\mu\nu}} Z_{\mu\nu\alpha} = 0. \tag{13} \]

The last formula shows that if \( F \) is an observable for the irreducible theory, it is observable also for the reducible one. Conversely, if \( \tilde{F}(\psi_{\mu\nu}) \) is an observable for the reducible model (i.e. \( \tilde{F} \) verifies (13)), then it remains so in the irreducible case because it automatically checks (11). In conclusion, the zeroth order cohomological groups associated with the irreducible, respectively, reducible BRST operator are equal. Moreover, the numbers of physical degrees of freedom corresponding to both cases are equal, therefore the path integrals associated with the irreducible and reducible systems describe the same theory.

It is well-known that the BRST construction relies on homological perturbation theory that requires the acyclicity of the Koszul-Tate operator, \( \delta \). For a given gauge theory, \( \delta \) can be recursively derived antighost level by antighost level. In our irreducible approach, the minimal ghost spectrum contains the bosonic ghosts \( \eta_\alpha \) with ghost number one, while the minimal antifield spectrum involves the fields

\[ \left( \psi^{*}_{\mu\nu}, \varphi^{*}, \eta^{*}_\alpha \right), \tag{14} \]
with the ghost numbers \((gh)\) and Grassmann parities \((\epsilon)\) expressed by

\[
gh \left( \psi_{\mu \nu}^* \right) = gh \left( \varphi^* \right) = -1, \quad gh \left( \eta_\alpha^* \right) = -2, \quad \epsilon \left( \psi_{\mu \nu}^* \right) = \epsilon \left( \varphi^* \right) = 0, \quad \epsilon \left( \eta_\alpha^* \right) = 1.
\]

(15) \hspace{1cm} (16)

We define the action of \(\delta\), as usually, through

\[
\delta \psi_{\mu \nu} = 0, \quad \delta \varphi = 0, \quad \delta \eta_\alpha = 0,
\]

(17)

\[
\delta \psi_{\mu \nu}^* = -\frac{\delta S^L_0}{\delta \psi_{\mu \nu}} , \quad \delta \varphi^* = -\frac{\delta S^L_0}{\delta \varphi} = 0,
\]

(18)

\[
\delta \eta_\alpha^* = 2 \left( \delta_{\alpha \beta} - \frac{1}{4} \gamma_\alpha \gamma_\beta \right) \partial_\mu \psi_{\mu \beta}^* + \partial_\alpha \varphi^*.
\]

(19)

The antifield \(\varphi^*\) being \(\delta\)-closed, it follows that there can be nontrivial co-cycles in the homology of \(\delta\) at non-vanishing resolution degrees. In order to show that \(\delta\) is however acyclic, we prove that \(\varphi^*\) is \(\delta\)-exact. Multiplying (19) from the left by \(\gamma_\alpha\), we find after simple computation

\[
\varphi^* = \delta \left( \frac{\delta}{\Box} \gamma_\alpha \eta_\alpha^* \right),
\]

(20)

hence \(\varphi^*\) is \(\delta\)-exact.

The last step of this treatment resides in deriving the path integral of the irreducible theory. With the above spectra at hand, the non-minimal solution of the master equation reads as

\[
S = S^L_0 + \int d^4 x \left( \bar{\psi}_{\mu \nu}^* \left( \delta_{\nu \beta} \partial_\mu + \delta_{\mu \beta} \partial_\nu \right) \left( \delta_{\beta \alpha} - \frac{1}{4} \gamma_\beta \gamma_\alpha \right) \eta_\alpha + \bar{\varphi}^* \partial_\alpha \eta_\alpha + \bar{b}_\alpha C^*_\alpha \right),
\]

(21)

where any bar variable denotes the conjugated of the corresponding field, and \((b_\alpha, b^*_\alpha, C_\alpha, C^*_\alpha)\) form the non-minimal sector.

We pass to the gauge-fixing procedure. For subsequent purpose, we will implement three gauge-fixing fermions. The first fermion is taken of the form

\[
K = \int d^4 x \bar{\chi}_\alpha \chi_\alpha.
\]

(22)
with
\[ \chi_\alpha = \gamma_\nu \psi_{\alpha \nu} - \frac{1}{4} \gamma_\alpha \psi_{\nu \nu} + \gamma_\alpha \varphi - \frac{1}{2} b_\alpha \equiv \rho_\alpha - \frac{1}{2} b_\alpha. \quad (23) \]

Eliminating the anti-fields from (21) with the aid of (22), we arrive at the gauge-fixed action
\[ S_K = S^L_0 + \int d^4x \left( \overline{\psi} \gamma_\alpha \partial_\mu \eta_\mu + b_\alpha \chi_\alpha + \frac{1}{2} \overline{\chi}_\lambda \left( \delta_\lambda_\mu \gamma_\nu + \delta_\lambda_\nu \gamma_\mu - \frac{1}{2} \delta_\mu_\nu \gamma_\lambda \right) \left( \delta_\nu_\beta \partial_\mu + \delta_\mu_\beta \partial_\nu \right) \left( \delta_\beta_\alpha - \frac{1}{4} \gamma_\beta \gamma_\alpha \right) \eta_\alpha \right). \quad (24) \]

Using the concrete form of (24), we can emphasize clearer the advantages of our irreducible procedure. The last term from (24) is invariant (as in the reducible case) under the gauge transformations
\[ \overline{\psi}_\lambda \to \overline{\psi}_\lambda + \gamma_\lambda, \quad \eta_\alpha \to \eta_\alpha + \gamma_\alpha \eta, \quad (25) \]

with \( \overline{\psi} \) and \( \eta \) arbitrary spinors. These invariances are however cancelled by the term \( \overline{\psi} \gamma_\alpha \partial_\mu \eta_\mu \), which simultaneously fixes \( \overline{\psi}_\lambda \) and \( \eta_\alpha \). In the reducible approach, there is necessary to supplement the ghost spectrum with ghosts of ghosts (and consequently enlarge the non-minimal sector) in order to fix the above invariances, in contrast with the present case. In order to make the link with the reducible approach exposed in [2], we choose an alternative gauge-fixing fermion under the form
\[ K' = a \int d^4x \overline{\psi} \phi \chi_\alpha, \quad (26) \]

with \( a \) a non-vanishing constant. In this case the corresponding gauge-fixed action is given by
\[ S_{K'} = S^L_0 + a \int d^4x \left( \overline{\psi} \phi \gamma_\alpha \partial_\mu \eta_\mu + b_\alpha \phi \chi_\alpha + \overline{\chi}_\lambda \phi \left( \delta_\lambda_\mu \gamma_\nu + \delta_\lambda_\nu \gamma_\mu - \frac{1}{2} \delta_\mu_\nu \gamma_\lambda \right) \left( \delta_\nu_\beta \partial_\mu + \delta_\mu_\beta \partial_\nu \right) \left( \delta_\beta_\alpha - \frac{1}{4} \gamma_\beta \gamma_\alpha \right) \eta_\alpha \right). \quad (27) \]

Eliminating the auxiliary fields \( b_\alpha \) on their equations of motion, we find
\[ S_{K'} = S^L_0 + a \int d^4x \left( \overline{\psi} \phi \gamma_\alpha \partial_\mu \eta_\mu + \frac{1}{2} \overline{\rho} \phi \rho_\alpha + \overline{\chi}_\lambda \phi \left( \delta_\lambda_\mu \gamma_\nu + \delta_\lambda_\nu \gamma_\mu - \frac{1}{2} \delta_\mu_\nu \gamma_\lambda \right) \left( \delta_\nu_\beta \partial_\mu + \delta_\mu_\beta \partial_\nu \right) \left( \delta_\beta_\alpha - \frac{1}{4} \gamma_\beta \gamma_\alpha \right) \eta_\alpha \right). \quad (28) \]
The last two terms in (28) are identical with the corresponding ones from [2], while the term \( C / @ \) replaces the remaining terms appearing in \( S_{\text{gauge}} \) and \( S_{\text{ghost}} \) derived within this reference. The “Nielsen-Kallosh ghost” for spin-5/2 gauge fields present in [2] is absent in our procedure, but the role of the extraghost \( C' \) is played here by \( \varphi \). The third gauge-fixing fermion to be discussed below allows us to make a proper correlation with the gauge-fixed action to be derived in the framework of the BRST-anti-BRST formalism (see the next section). It has the expression

\[
K'' = \int d^4 x \mathcal{C}_{\alpha} \psi_{\alpha},
\]

with

\[
\psi_{\alpha} = 2 \left( \delta_{\alpha\beta} - \frac{1}{4} \gamma_\alpha \gamma_\beta \right) \partial_\beta \varphi + \partial_\alpha \psi_{\lambda\lambda} + \bar{b}_\alpha.
\]

The resulting gauge-fixed action is

\[
S_{K''} = S_0' + \int d^4 x \left( -2 \left( \partial_\alpha \mathcal{C}_{\alpha} \right) \left( \delta_{\beta\lambda} - \frac{1}{4} \gamma_{\beta\lambda} \right) \partial_\beta \eta_{\lambda} - 2 \left( \partial_\beta \mathcal{C}_{\alpha} \right) \left( \delta_{\beta\alpha} - \frac{1}{4} \gamma_{\alpha\beta} \right) \partial_\lambda \eta_{\lambda} + \bar{b}_\alpha \psi_{\alpha} \right). \tag{31}
\]

The gauge conditions implemented via (30) have been not used so far in the literature. Nevertheless, (30) stand for some good canonical gauge conditions because they lead to the terms

\[
\frac{1}{2} \int d^4 x \left( \bar{\psi}_{\mu\nu} \psi_{\nu\nu} + \frac{3}{2} \bar{\varphi} \varphi + \bar{\psi}_{\mu\nu} \bar{b} \right), \tag{32}
\]

in the gauge-fixed action after eliminating the auxiliary fields \( b_\alpha \) (on their field equations). All the terms in (32) are linear in the derivatives, as requested by field theories with fermions in order to prevent the existence of negative-norm states.

4 The irreducible BRST-anti-BRST procedure

Here we develop the antifield BRST-anti-BRST quantization of action (10), subject to the irreducible gauge transformations (3) and (9). In connection
with the general approach of the antifield BRST-anti-BRST treatment, we follow the line from \cite{3}, \cite{10}. However, the ideas from \cite{3}, \cite{10} are not enough in the context of our irreducible procedure. They have to be supplemented with the analysis from the beginning of section 3. The field, respectively, ghost spectra read as

\begin{equation}
(0,0) \quad \begin{pmatrix} \psi_{\mu\nu} \end{pmatrix} ; \varphi, \quad (33)
\end{equation}

\begin{equation}
(1,0) \quad \begin{pmatrix} \eta_1 \alpha, \eta_2 \alpha, \pi_\alpha \end{pmatrix}, \quad (34)
\end{equation}

while the antifield spectrum is given by

\begin{equation}
(0) \quad \begin{pmatrix} \psi^*_{\mu\nu} ; \varphi^* ; \psi^*_{\mu\nu} ; \varphi^* \end{pmatrix}, \quad (35)
\end{equation}

\begin{equation}
(1) \quad \begin{pmatrix} \eta_1^* \alpha, \eta_2^* \alpha, \pi^*_1 \alpha, \pi^*_2 \alpha, \psi^{(B)} ; \varphi^{(B)} \end{pmatrix}, \quad (36)
\end{equation}

\begin{equation}
(2) \quad \begin{pmatrix} \eta_1^{(B)} \alpha, \eta_2^{(B)} \alpha \end{pmatrix}, \quad (37)
\end{equation}

In (33)-(37), the superscript \((a,b)\) denote the bighost bidegree, the notation \(F^{(B)}\) signifying the bar variable corresponding to \(F\), in order to avoid confusion with the operation of spinor conjugation. In the sequel we will omit the superscript for simplicity. With the help of the above spectra, we derive the solution of the master equation in the BRST-anti-BRST formalism under the form

\begin{equation}
\bar{S} = S^L_0 + \int d^4x \left( \bar{\psi}_{\mu\nu}^* (\delta_{\nu\beta} \partial_\mu + \delta_{\mu\beta} \partial_\nu) \left( \delta_{\beta\alpha} - \frac{1}{4} \gamma_{\beta\gamma\alpha} \right) \eta_1 \alpha + \right.

\begin{equation}
\bar{\psi}_{\mu\nu}^* (\delta_{\nu\beta} \partial_\mu + \delta_{\mu\beta} \partial_\nu) \left( \delta_{\beta\alpha} - \frac{1}{4} \gamma_{\beta\gamma\alpha} \right) \eta_2 \alpha + \varphi^* (\delta_{\beta\alpha} - \frac{1}{4} \gamma_{\beta\gamma\alpha} \pi_\alpha + \varphi^{(B)} \partial_\alpha \pi_\alpha + \left( \eta_1^{(B)} \alpha - \eta_2^{(B)} \alpha \right) \pi_\alpha \right).
\end{equation}

In order to fix the gauge, we introduce the variables \cite{3}, \cite{10}

\begin{equation}
(0) \quad \begin{pmatrix} \psi \end{pmatrix} ; \varphi \quad (38)
\end{equation}

\begin{equation}
(1) \quad \begin{pmatrix} \mu_{(1)\mu\nu} ; \mu_{(1)\alpha} ; \mu_{(1)\alpha} ; \mu_{(1)\alpha} \end{pmatrix}, \quad (39)
\end{equation}
and consider the new solution

\[ S_1 = \tilde{S} + \int d^4x \left( \bar{\psi}^{*(2)} \mu^{(ψ)}_{(1)μν} + \bar{ϕ}^{*(2)} \mu^{(ϕ)}_{(1)} + \bar{η}_{2α}^{*(2)} \mu^{(η)}_{(1)α} + \bar{π}^{*(2)}_{α} \mu^{(π)}_{(1)α} \right). \]  

(40)

With the help of the previous solution, we can fix the gauge taking the gauge-fixing boson

\[ F = \int d^4x \left( \bar{ψ}_{μν}γ_μγ_νϕ + \bar{η}_{1α}θνη_{2α} \right). \]  

(41)

Eliminating from (40) the bar variables (those carrying the index \((B)\)), and the anti-fields with the index \((1)\) in the usual way \([8, 10]\), we obtain the gauge-fixed action

\[ \tilde{S}_{1F} = S_0^L + \int d^4x \left( 2\bar{μ}^{(ψ)}_{(1)} \left( \delta_βγ_α - \frac{1}{4}γ_βγ_α \right) \partial_βη_{1α} + \bar{μ}^{(η)}_{(1)μ} \partial_μη_{1α} + \bar{π}^{(π)}_{α} \mu^{(π)}_{(1)α} \right) + \bar{η}_{2α}^{*(2)} \mu^{(η)}_{(1)α} + \bar{π}^{*(2)}_{α} \mu^{(π)}_{(1)α}. \]  

(42)

We further eliminate the auxiliary anti-fields with the index \((2)\), and the \(μ\)'s from (42) on their equations of motion, arriving at

\[ \tilde{S}_{1F} = S_0^L + \int d^4x \left( -2 \left( \partial_β\bar{η}_{1α} \right) \left( \delta_αβ - \frac{1}{4}γ_αγ_β \right) \partial_μη_{2μ} - 2 \left( \partial_μ\bar{η}_{1μ} \right) \left( \delta_αβ - \frac{1}{4}γ_αγ_β \right) \partial_βη_{2α} + \bar{π}_α \left( 2 \left( \delta_αβ - \frac{1}{4}γ_αγ_β \right) \partial_βϕ + \partial_αψ_{μμ} + ϕ_πα \right) \right). \]  

(43)

This is the final result of our irreducible anti-field BRST-anti-BRST formalism. The gauge-fixed action (43) is identical with (31), modulo the identifications

\[ \bar{η}_α = \bar{η}_{1α}, \ \bar{η}_α = \bar{η}_{2α}, \ \bar{b}_α = \bar{π}_α. \]  

(44)
5 Conclusion

We showed that free massless spin-$5/2$ gauge fields can be consistently quantized as an irreducible theory within both the antifield BRST and BRST-anti-BRST approaches. In this context, although the starting model is reducible, the ghosts of ghosts are not necessary. This is because we replace the initial reducible generating set with an irreducible one, such that the physical observables remain the same in both formulations. The irreducibility is gained by introducing a spin-$1/2$ gauge field having trivial field equation. The triviality of the spin-$1/2$ gauge field equation implies the $\delta$-closedness of the associated antifield. In spite of this, the irreducible Koszul-Tate operator is proved to be truly acyclic at non-vanishing antighost numbers. In the framework of the BRST procedure we discuss three possibilities of fixing the gauge. The first one emphasizes in a clearer fashion the meaning of our irreducible treatment. The second choice is helpful at establishing a comparison with the reducible methods employed in the literature with regard to the investigated model. The third election is taken in order to make manifest the equivalence with the BRST-anti-BRST gauge-fixed action.

References

